Name: ____ Period: ______ Date: ___________

Honors: Finding turning points of a polynomial

We've been locating zeros for polynomial functions by factoring. For example:

$$
\begin{aligned}
&f(x)=2x^2+x-15\\
&a\cdot c=-30, b=1 \quad \rightarrow \quad b_1=6, b_2=-5\\
&f(x)=2x^2+6x-5x-15\\
&f(x)=2x(x+3)-5(x+3)\\
&f(x)=(2x-5)(x+3)\\
&\therefore \text{Zeros at } x=5/2, -3
\end{aligned}
$$

However, if you want to find the <u>*turning points*</u> (which represent *maximums* and *minimums*), factoring the function won't help you! I'm going to teach you a trick from Calculus to find the turning points. From your starting function, *f*, you need to create a new function, f' ("f prime"). The new function is called the **<u>derivative</u>** of *f*. Look at these examples to see if you can find the pattern for taking the derivative of a polynomial function:

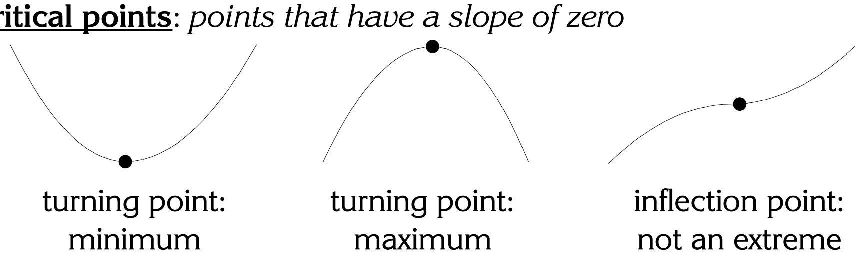
$$f(x)=x^3+x^2+x \rightarrow f'(x)=3x^2+2x+1$$
$$f(x)=3x^5-4x^2+7x \rightarrow f'(x)=15x^4-8x+7$$
$$f(x)=x^2+5x+9 \rightarrow f'(x)=2x+5$$
$$f(x)=20 \rightarrow f'(x)=0$$
$$f(x)=-2x^{10}+5x^2-13x+88 \rightarrow f'(x)=-20x^9+10x-13$$

The derivative is a way to calculate the **<u>slope</u>** of your original function at any *x* you want. For example, $L(x)=6x+10$ is a linear function with slope 6. Because it's linear, the slope of the graph is <u>always</u> 6, no matter what value of *x* you check. The derivative is $L'(x)=6$. So, you can see that no matter what *x* you check, $L'(x)$ always tells you "the slope of *L* there is 6".

A parabola has a slope which changes depending where you look. One of the examples above shows the parabola $f(x)=x^2+5x+9$, with a derivative of $f'(x)=2x+5$. The slope of this parabola at $x=7$ is therefore $f'(7)=2\cdot7+5=19$. The slope at $x=-3$ is $f'(-3)=2\cdot-3+5=-1$. The slope at $x=-5/2$ is $f'(-5/2)=2(-5/2)+5=0$. And so on!

Why is this useful? Turning points always have a slope of zero, because the graph has reached either a maximum or minimum value: there's a tiny "flat" spot there. So, if you *find the zeros of your derivative*, $f'(x)$, you can find all the "flat" spots on $f(x)$, known as **<u>critical points</u>**. For the parabola in the example above, $x=-5/2$ is a critical point.

<u>Critical points</u>: *points that have a slope of zero*



An **<u>inflection point</u>** is a spot where the graph flattens out for a moment, but then keeps going without turning around.

(You might have fun graphing some of the example functions and their derivatives together.)

1) Consider $f(x)=16x^2-12$.

A) Find the derivative, $f'(x)$.

B) Find the zeros of $f'(x)$. You should find one rational answer.

C) Based on its leading term, what is the end behavior of *f*(*x*)? Do you think the turning point you found is a maximum or a minimum? (You should be able to answer this without actually graphing anything, but if you're desperate, go ahead and graph it.)

2) Consider $f(x)=2x^3+x^2-4x$.

A) Find the derivative, $f'(x)$.

B) Find the zeros of $f'(x)$. You should find two rational zeros.

C) Based on the end behavior of *f*(*x*), which zero is a maximum and which is a minimum?

3) Consider $f(x)=-3x^3+4x^2+2x+15$.

A) Find the derivative, $f'(x)$.

B) Find the zeros of $f'(x)$. The zeros this time will be irrational. You'll need to use the quadratic formula to find them, and the answers will still contain a radical sign. (" $\sqrt{\ }$ ")

C) Which is a maximum and which is a minimum?