Reference: Function Transformations

Outside vs. Inside

- Changes “outside” of the function's argument...
  - example: $g(x) = f(x) + 3$
  - cause VERTICAL (up/down) changes
  - affect the RANGE (*y* values) of the function
  - to find the new rule, replace “f(x)” with the expression for f(x)
- Changes “inside” the function's argument...
  - example: $h(x) = f(x+5)$
  - cause HORIZONTAL (left/right) changes
  - affect the DOMAIN (*x* values) of the function
  - to find the new rule, replace “x” in f(x) with whatever you see in the parentheses

Types of Transformation

- Translations - “slide” the function without changing its shape
  - $g(x) = f(x) + k$ ... $\uparrow$ ... move up *k*
  - $g(x) = f(x) - k$ ... $\downarrow$ ... move down *k*
  - $g(x) = f(x-h)$ ... $\rightarrow$ ... move right *h* (function “starts later”)
  - $g(x) = f(x+h)$ ... $\leftarrow$ ... move left *h* (function “starts earlier”)
- Stretches and Compressions – deform the function
  - $g(x) = a \cdot f(x)$ with $a > 1$ ... $\updownarrow$ ... stretch vertically by factor of *a* (“taller”)
  - $g(x) = a \cdot f(x)$ with $0 < a < 1$ ... $\downarrow\uparrow$ ... compress vertically by factor of *a* (“shorter”)
  - $g(x) = f(b \cdot x)$ with $0 < b < 1$ ... $\leftrightarrow$ ... stretch horiz. by factor of *b* (“slower” / “wider”)
  - $g(x) = f(b \cdot x)$ with $b > 1$ ... $\rightarrow\leftarrow$ ... compress horiz. by factor of *b* (“faster” / “narrower”)
- Reflections – flip the function over
  - $g(x) = -f(x)$ ... ↶ ... vertical reflection / reflect across the *x*-axis
  - $g(x) = f(-x)$ ... ↷ ... horizontal reflection / reflect across the *y*-axis

Piecewise Transforms

- For an “outside” transformation, transform the rules of the function. You can leave the domains alone because outside transformations only affect the range.
- For an “inside” transformation, transform the rules AND the domains.

Examples

$\boldsymbol{f(x) = 2x + 4}$
$\boldsymbol{g(x) = f(x) + 3}$
$g(x) = (2x+4) + 3$
$g(x) = 2x + 7$

$\boldsymbol{f(x) = 2x + 4}$
$\boldsymbol{h(x) = f(x+5)}$
$h(x) = 2(x+5) + 4$
$h(x) = 2x + 10 + 4$
$h(x) = 2x + 14$

$\boldsymbol{f(x) = 2x + 4}$
$\boldsymbol{j(x) = f(x+5) + 3}$
$j(x) = (2(x+5) + 4) + 3$
$j(x) = (2x + 10 + 4) + 3$
$j(x) = (2x + 14) + 3$
$j(x) = 2x + 17$